Mesh Sensitivity Analysis Using Richardson Extrapolation in a Rayleigh-Benard Convection Problem

Omar Rahli
Laboratory of Polyphasic Transport and Porous Media, University of Science and Technology Houari Boumediene, Algeria

Najwa Mimouni
Laboratory of Polyphasic Transport and Porous Media, University of Science and Technology Houari Boumediene, Algeria

Abstract

There are several modes of heat transfer in our universe, and their significance has grown in recent years due to increasing energy demands. This work focuses on natural convection, a phenomenon commonly found in both industrial and natural settings. The study of heat transfer by natural convection in porous media has attracted considerable attention from researchers and remains a current topic of interest. This is largely due to its wide range of practical applications, such as geothermal energy extraction, oil recovery, heat exchangers, and agricultural product storage.

Various studies on this subject encompass experimental investigations, analytical solutions, and numerical modeling. The introduction of a porous medium aims to enhance heat transfer and thus maximize energy efficiency. Research has been conducted on the different types of flow that can occur in porous cavities under steady-state conditions, which generally depend on several factors, including fluid storage properties, transfer characteristics, and mechanical properties.

While convection as a heat transfer mechanism was first identified by Rumford in 1797, a comprehensive theoretical understanding was only developed later, notably by Lord Rayleigh in 1916. Earlier experimental work by Bénard (1900–1901) on a free liquid layer-although primarily highlighting surface tension effects rather than thermal convection—laid the groundwork for the first theoretical models, including that of Boussinesq in 1903, and subsequently Rayleigh's more complete formulation. Rayleigh demonstrated that convection within a fluid arises when the ratio between buoyancy-driven forces (which promote motion) and viscous and thermal diffusion forces (which oppose it) exceeds a critical threshold. This dimensionless ratio, known as the Rayleigh number, must surpass a critical value for the onset of natural convection to occur.

Our study focuses on the numerical investigation of natural convection within a porous domain under Rayleigh-Bénard instability. The problem is formulated under the assumption of laminar flow, using the Darcy-Brinkman model and the Boussinesq approximation. The governing equations are discretized using the finite volume method and solved numerically through an in-house computational code based on the SIMPLER algorithm, which is employed to handle the velocity-pressure coupling.

For the selected range of control parameters, we present the behavior of flow structures and heat transfer for both a fully fluid cavity and a partially porous enclosure. The second part of the study is devoted to the extrapolation of the converged solutions obtained with different mesh sizes toward a more accurate result using Richardson extrapolation. This part analyzes the effectiveness of applying Richardson's algorithm to enhance the numerical solution while optimizing mesh sensitivity and reducing computational cost. We have demonstrated that the Richardson extrapolation-based algorithm yields very satisfactory results, particularly in the case involving three mesh levels. However, the extrapolation does not ensure convergence throughout the entire computational domain, mainly due to the discretization scheme used for the advection terms. The next step will focus on incorporating high-order numerical schemes to enhance the overall convergence rate.

Keywords

CFD, Porous media, Rayleigh-Bénard Convection, Richardson Extrapolation.