Estimation of Distributionally Ordered Survival Functions under Censoring and Truncation

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Abstract

A nonparametric maximum likelihood procedure proposed by Turnbull (1976) for estimating distribution functions with arbitrarily grouped, censored, and truncated data is extended to accommodate general distributional constraints in a unified framework. The method retains the nonparametric nature of Turnbull's original algorithm while incorporating inequality-type restrictions through well-studied constrained optimization techniques based on Lagrange multipliers and product- multinomial likelihoods (Luenberger, 1969; Rockafellar, 1972). A two-step constrained Turnbull procedure is developed, in which each iteration alternates between a Turnbull updating step and a constraint step that can exploit existing results for order-restricted inference (Barlow et al., 1972; Barlow and Brunk, 1972; Robertson and Wright, 1981; Robertson et al., 1988; Dykstra, 1982; Feltz and Dykstra, 1985; Dykstra et al., 1991, 1995). Applications are given to several forms of distributional ordering for survival data, including standard stochastic ordering, uniform stochastic ordering (Dykstra et al., 1991), and likelihood ratio ordering (Dykstra et al., 1995), all in the presence of censoring and truncation. The methodology is illustrated using a retrospective study of time to cosmetic deterioration in women with early breast cancer treated either with radiotherapy alone or with radiotherapy plus adjuvant chemotherapy (Beadle et al., 1984a, 1984b; Klein and Moeschberger, 1997), where constrained nonparametric MLEs of survival functions are shown to respect the assumed orderings while remaining close to the corresponding unconstrained estimates.

Keywords

Constrained Nonparametric Maximum Likelihood Estimation, Censored Data, Truncated Data, Distributional Ordering, Stochastic Ordering, Uniform Stochastic Ordering, Likelihood Ratio Ordering, Survival Function.