The Følner Sequence of the Baumslag-Solitar Groups BS(1,n)

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Abstract

The Baumslag-Solitar groups BS(1,n) defined by

BS(1,n) = <a,b | a-1ba = bn>

are metaabelian, i.e., they are extensions of abelian groups by abelian groups. Such groups are in particular amenable. Therefore one of the equivalent conditions of amenability is met: the Følner condition:

$\forall \varepsilon>0, \exists F \subset BS(1,n): \sharp F < \infty \ \& \ (\sharp \partial F)/(\sharp F)< \varepsilon$

where $\sharp F$ is the number of elements of a set F and ∂F is the outer boundary of F defined by

$\partial F = \{g \in BS(1,n) \setminus F \mid \exists f \in F \ \& \ \exists t \in \{a,b,a^{-1},b^{-1}\}: tf = g\}$

An interesting question is for every positive integer k to find the number of elements Føl(k) of the smallest set F_k that satisfies

$(\sharp \partial F\_k)/(\sharp F\_k) \leq 1/k$