Geometric Means of HPD GLT Matrix-Sequences: Beyond Invertibility Assumptions and Convergence Properties

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Abstract:

In this work, we extend our study on the spectral distribution of the geometric mean of matrix-sequences formed by Hermitian Positive Definite (HPD) matrices, assuming that all input matrix-sequences belong to the same Generalized Locally Toeplitz (GLT) *-algebra. Building upon our previous results [1], we further examine whether the assumption that at least one of the input GLT symbols is invertible almost everywhere is necessary. Since inversion is mainly required due to non-commutativity matrices, we aim to investigate whether this assumption can be removed when the GLT symbols commute for every $d,r\geq 1$. Numerical experiments are conducted to support this idea. Additionally, we investigate the Karcher mean of more than two HPD GLT matrix-sequences and study its GLT nature. In particular, we analyze how starting with an initial guess that is already a GLT matrix-sequence affects the convergence of the iterative process. Our goal is not just to compute the geometric mean, but to observe how quickly the iterations reach the final result when a suitable initial guess is used. The study also extends the proof for the block multilevel case ($r=1$ and $d\geq 1$), while refining numerical validation in broader settings.

Keywords:

Geometric Mean, Matrix-Sequences, Karcher Mean, Generalized Locally Toeplitz (GLT) Algebra, Spectral Distribution.