

Hamilton-Jacobi Formalism for Thermodynamics: A Detailed Theoretical and Applied Study

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Abstract

We systematically develop the Hamilton-Jacobi (HJ) formalism for thermodynamics, motivated by its success in classical mechanics and optics. By reformulating the laws of thermodynamics in a geometric framework using contact manifolds and Legendre transforms, we demonstrate that familiar equations of state arise as solutions to the HJ equation. Detailed applications are provided for the ideal gas, van der Waals gas, and Curie-Weiss magnet, with each example illuminating the power and interpretation of the method. We discuss the conceptual meaning of “thermodynamic time” and the advantages of a unified geometric viewpoint, and propose directions for future research in generalized and quantum thermodynamics.

Keywords

Hamilton-Jacobi, thermodynamics, Legendre transform, contact geometry, equations of state, geometric thermodynamics, phase space, statistical mechanics.