Growth Properties of Solutions to Specific Classes of Complex Differential Equations

Benharrat Belaidi
Department of Mathematics, Laboratory of Pure and Applied Mathematics, University of Mostaganem, B. P 227 Mostaganem, Algeria

Abstract

This talk investigates the growth and oscillatory properties of differential polynomials generated by solutions of a class of second-order non-homogeneous linear differential equations of the form

$$f'' + P(z)e^{a_n z^n} f' + B(z)e^{b_n z^n} f = F(z)e^{a_n z^n}$$

where a_n, b_n are complex constants $P(z)$ is a nontrivial polynomial, and $B(z)$ and $F(z)$ are nontrivial entire functions of order strictly less than n. The structure of the equation, characterized by exponential factors involving powers of z, introduces significant complexity in understanding the asymptotic behavior of its solutions.

The main focus is on how the growth of solutions influences the behavior of associated differential polynomials. By applying techniques from complex differential equations and Nevanlinna theory, the study derives new estimates for the hyper-order of solutions and related expressions. These estimates provide refined information about how rapidly solutions and their derivatives grow in comparison with the exponential coefficients present in the equation.

In addition, the talk examines the oscillation of solutions and the associated differential polynomials by analyzing the distribution of their zeros and fixed points. Conditions are established under which both the solutions and the corresponding differential polynomials exhibit oscillatory behavior, as well as criteria ensuring that they have infinitely many fixed points. The interplay between the coefficients $P(z)$, $B(z)$, and $F(z)$ and the exponential terms plays a crucial role in determining these qualitative features.

The results extend and complement existing work on homogeneous and previously studied equations by addressing the combined effects of non-homogeneity and exponential growth factors. This presentation is based on recent results obtained by the author, as presented in Reference [5]. Overall, these findings contribute to a deeper understanding of the relationship between the analytic properties of the coefficients and the global behavior of solutions, offering new insights into the dynamics of complex differential equations.