On Solvability of the Nonlinear Biharmonic Equation Subjected to the Swell Pressure

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Abstract

This work focuses on the flexure of thin plates caused by expansive soil movements [1, 2]. The existing experimental evidence suggests that this expansive soil poses a significant threat to underground thin plates [3].

We state well-known equation of the small deflection of thin plates

$$\frac{\partial^2}{\partial x^2}\left[D\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)\right]+ 2(1-\nu)\frac{\partial^2}{\partial x\partial y}\left(D\frac{\partial^2 w}{\partial x\partial y}\right)+\frac{\partial^2}{\partial y^2}\left[D\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)\right]-q=0, \tag{1}$$

where ν is Poisson's ratio $\left(0 < \nu < \frac{1}{2}\right)$, *q* is the distributed load, and *D* is flexural rigidity of the plate.

So, by using equation (1), we consider the following BVP

$$\Delta^2 w = Q_g(x,y) - Q_s(x,y), \qquad \forall (x,y) \in \Omega \subset \mathrm{R}^2, \tag{2}$$

$$w|_{\partial\Omega} = 0, \qquad \frac{\partial}{\partial \vec{n}} w|_{\partial\Omega} = 0, \tag{3}$$

where $Q_g(x,y)$ is the distributed weight load, $Q_s(x,y)$ is the distributed load of expansive soil.

We denote by $w_0(x,y)$ the solution of the linear problem

$$\Delta^2 w_0 = Q_s(x,y), \qquad \forall (x,y) \in \Omega \subset \mathrm{R}^2, \tag{4}$$

$$w_0|_{\partial\Omega} = 0, \qquad \frac{\partial}{\partial \vec{n}} w_0|_{\partial\Omega} = 0. \tag{5}$$

It is well-known [4] that there exists a unique solution of the problem (4), (5).

Let $\tilde{v}(x,y) = w(x,y) - w_0(x,y)$ $\forall (x,y) \in \Omega$. Then $v(x,y)$ is the solution of the following nonlinear boundary value problem

$$\Delta^2 v = \alpha k(x,y) e^{-v(x,y)}, \qquad \forall (x,y) \in \Omega \subset \mathrm{R}^2, \tag{6}$$

$$v|_{\partial\Omega} = 0, \qquad \frac{\partial}{\partial \vec{n}} v|_{\partial\Omega} = 0, \tag{7}$$

where $v(x,y) = \alpha \tilde{v}(x,y)$, $k(x,y) = \sigma_0 D_p e^{-\alpha w_0(x,y)}$ with α is some physical constant.

We denote by $C^{(4)}(\Omega)$ the set of functions $u(x,y)$ whose partial derivatives up to order four are all continuous on Ω. And $C^{(4)}_{w_0}(\Omega)$ denotes the set of functions $v(x,y) \in C^{(4)}(\Omega)$ that satisfy inequality $0 \le v(x,y) \le w_0(x,y)$ for all $(x,y) \in \Omega$.

Furthermore, only the circular plate is studied, i.e. $\Omega = \{(x,y) \in \mathrm{R}^2 : x^2 + y^2 < 1\}$.

Theorem: *There exists a unique solution $v(x,y)$ of the nonlinear problem (6), (7) in $C^{(4)}_{w_0}(\Omega)$.*

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Keywords

Existence, uniqueness, swell pressure, biharmonic equation.