Energy-Based Modeling and Structure-Preserving Discretization of Physical Systems

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Abstract

This paper develops a comprehensive mathematical framework for energy-based modeling of physical systems, with particular emphasis on preserving fundamental structural properties throughout the modeling and discretization process. The approach provides systematic methods for handling challenging system classes including high-index differential-algebraic equations and nonlinear multiphysics problems. Theoretical foundations are established for regularizing constrained systems while maintaining physical consistency, analyzing stability properties, and constructing numerical discretizations that inherit the energy dissipation structure of the continuous models. The versatility and practical utility of the framework are demonstrated through applications across multiple domains including poroelastic media, nonlinear circuits, constrained mechanics, and phase-field models. The results ensure that essential physical properties such as energy balance and dissipation are maintained from the continuous formulation through to numerical implementation, providing robust foundations for computational physics and engineering applications.

Keywords

Energy-based modeling, Port-Hamiltonian systems, Structure-preserving discretization, Differential-algebraic equations, Discrete gradient methods, Energy dissipation, Exponential stability, Poroelasticity, Nonlinear circuits.