Symmetrizability Through Involutory Matrices

Gunjan Agrawal
Department of Mathematics, Faculty of Science, Dayalbagh Educational Institute (Deemed to be University), Dayalbagh, Agra

Shinu Pachauri
Department of Mathematics, Faculty of Science, Dayalbagh Educational Institute (Deemed to be University), Dayalbagh, Agra

Abstract

Symmetric systems help one understand the dynamics of Hessian geometrical models in Solid Mechanics, damped oscillations in Classical Mechanics and conservative systems in Dynamical Systems in a systematic manner. Efforts have been made since long to convert asymmetric systems into equivalent symmetric systems to take advantage of the well-developed theories for symmetric systems. In this context, Taussky in 1968 defined a matrix to be symmetrizable if it is similar to its transpose. Adhikari in 2010 generalized the definition by redefining and calling it the symmetrizability of second kind. In 2020, McKee and Smyth considered those symmetrizable matrices that are a real diagonal change of basis away from being symmetric and studied quotients of equitable partitions of graphs, examples and necessary combinatorial of SIMs (symmetrizable integer matrices with all non-negative entries) and the trace problem of positive algebraic integer symmetrizable matrices. In the present paper, the notion of I-symmetrizable matrices is introduced and studied. A matrix is defined to be I-symmetrizable if it is similar to a non-diagonal symmetric matrix through an involutory matrix. Classifying involutory matrices of order 2, involutory matrices of all higher orders are studied. Furthermore, sufficient conditions for I-symmetrizability of upper triangular matrices have been obtained. Present study is applied to the development of a novel social media platform, distinct from the existing ones, which facilitates two-way interaction.