The Integral Problem for Nonlocal Schrödinger Equations and Applications

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Abstract:
Here, the integral problem for linear and nonlinear nonlocal Schrˆdinger equations are studied. The equation involves a convolutions terms with a general kernel coe¢ cients whose Fourier transform are functions that satisÖes some growth conditions. By assuming on the integral condition and enough smoothness of the coe¢ cients functions, the local and global existence and uniqueness of solutions are established. We can obtain a di§erent classes of integral problems for nonlocal Schrˆdinger equations by choosing the space H and linear operators, which occur in a wide variety of physical systems.The aim here, is to study the existence, uniqueness and regularity properties of solution of the integral problem (IP) for nonlocal nonlinear Schrˆdinger equat¨on (NSE),

$$i\partial_t u + a * \Delta u + A * u = \Delta \left[B * F(u)\right], \, t \in (0, T), \, x \in \mathbb{R}^n, \qquad (1.1)$$

$$u(x, 0) = \varphi(x) + \int_0^T g(\sigma) u(x, \sigma) \, d\sigma \text{ for a.e. } x \in \mathbb{R}^n, \qquad (1.2)$$

where A = A (x), B = B (x) are linear and nonlinear operator functions in a Hilbert space H, respectively, a, g are complex valued functions, T ∈ (0, ∞], F (u) is a given nonlinear function and' (x) is a given H-valued functions. For g (σ) Ξ 0 the integral problem (1.1)—(1.2) it becomes the usual Cauchy problem for abstract Schrödinger equation (SE) (1.1).

Remark 1.1. Note that particularly, by choosing (σ) as a piecewise con-tinuous function on (0, T), the condition (1.2) can be expressed as the following multipoint nonlocal condition in time

$$u(x, 0) = \varphi(x) + \sum_{k=1}^{m} \alpha_k u(x, \lambda_k),$$

where m is a positive integer, k are complex numbers and k 2 (0; T):

The G^p-regularity properties (1.1)—(1.2) depends crucially on the presence of suitable kernel. Then the question that naturally arises is which of the possible forms of the operator kernel functions are relevant for the global well-posedness of (1.1) — (1.2). In this study, as a partial answer to this question, we derive the G^p-well posedness of the corresponding linear problem (here, s denotes the set of complex numbers). By chousing the space H and operators A, B, we obtain a different classes of nonlocal SEs which occur in application. Let we put H = l_2, choose A, B as infinite many matrices $[a_{mj}]$ and $[b_{mj}]$, respectively for m, j = 1, 2, ..., N, N ∈ ℕ, where ℕ-denote the set of natural numbers. Then we obtain the existence, uniqueness and regularity properties of IVP for infinity many system of nonlocal SEs

$$i\partial_t u_m + a\Delta u_m + \sum_{j=1}^{N} a_{mj} * u_m = \tag{1.3}$$

$$\sum_{j=1}^{N} \Delta b_{mj} u_m * F_m (u_1, u_2, ..., u_m), \ t \in [0, T], \ x \in \mathbb{R}^n,$$

$$u_m (x, 0) = \varphi_m (x) + \int_0^T g(\sigma) u_m (x, \sigma) d\sigma, \ m = 1.2, ..., N,$$

where $a_{mj} = a_{mj} (x)$, $b_{mj} = b_{mj} (x)$ are complex valued functions, 'm (x) are given data functions, f_m are nonlinear functions and $u_j = u_j (x, t)$.

Keyword:
Nonlocal equations, di§usion equations, Schr˘dinger equations, abstract di§erential equations, Fourier multipliers.