A System of Second-Order Differential Equations with Exponential Nonlinearity

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Abstract

The author considers equations of type

$$\begin{pmatrix} y_1 \\ \cdots \\ y_n \end{pmatrix}^{\bullet\bullet} = M \cdot \begin{pmatrix} \exp y_1 \\ \cdots \cdot \cdots \\ \exp y_n \end{pmatrix} \qquad (1)$$

where *M* is a constant symmetric real n×n matrix, while the dots in the superscripts or over symbols denote time differentiation.

For *n*=3 and a special *M*, the equations describe evolution of (appropriately transformed) length scales in a rapidly exploding universe, in a neighborhood of its primordial singularity. They can also describe many other systems, from nonlinear crystal lattices to thermodynamics.

System (1) is Hamiltonian. The corresponding quadrics of kinetic energy prove to constitute a useful tool for its geometric illustration and description. The evolution described by (1) strongly depends on the signature of the matrix *M*, which manifests itself in the different shapes of the quadrics.

An explicit exact solution of (1) is found for all *n* and all matrices *M*. It is tested for stability under small perturbations; a stability-determining matrix may be constructed out of *M*. The system is found to be non-integrable, with a few exceptions (e.g. the Toda lattice), despite having the exact solution.

An analysis is performed for non-exact solutions; among the results it has been found that the dependent variables (and the physical system) may undergo oscillations of approximate sawtooth shape.

Keywords

Expansion, Primordial singularity, Stability, Integrability, Oscillations.