Solitary and Traveling Wave Solutions of the Nonlinear Schrödinger Equation with Quadratic-Cubic-Quartic Nonlinearity

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Abstract:

The stabilization of 3-dimensional (3D) solutions to the nonlinear Schrödinger equation (NLSE) with a cubic (third-order, i.e. Kerr) nonlinearity is an important topic of ongoing research in the field of nonlinear optics [1]. The 3D solutions, unlike in the 1D case, are unstable due to critical and supercritical collapse [1]. One of the most common methods to introduce stability in the system is to include competing nonlinearities and the most studied such system has been the NLSE with the cubic-quintic nonlinearity [2]. Recently, a promising approach has been to introduce the Lee-Huang-Yang (LHY) correction [3] to the Gross-Pitaevskii equation, a variant of the NLSE describing Bose-Einstein condensates which includes an external potential [4]. The LHY correction applied to the NLSE adds a quartic (fourth-order) nonlinear term in the the 3D and the reduced 2D case [5] and a quadratic (second-order) nonlinear term in the 1D case [6]. Systems with quadratic and quartic nonlinearities exist in nonlinear optics [7].

In this paper we will combine the effects to find exact solutions to the NLSE with distributed coefficients and a quadratic-cubic-quartic nonlinearity by generalizing the Jacobi elliptic function (JEF) expansion method developed in [8]. The matching condition for the fourth-order nonlinearity and the second order transverse derivative gives the maximum degree of the JEF to be *2/3* in the ansatz and the principle of harmonic balance gives us solutions which also include the degrees of the JEF of *0* and *-2/3*. Both solitary and traveling wave solutions are obtained, as well as the so-called chirped solutions, which contain a quadratic dependence in the phase with respect to the transverse variables [8].